PHY5210 **Problem Set #2** **due 28 January 2015**

1. **(15 points):** Consider a frictionless puck on a horizontal turntable that is rotating counterclockwise with angular velocity Ω. (a) Write down Newton's second law for the coordinates x' and y' of the puck as seen by me standing on the turntable. (Be sure to include the centrifugal and Coriolis forces, but ignore the earth's rotation.) (b) Solve the two equations by the trick of writing $\eta' = x' + iy'$ and guessing a solution of the form $\eta' = e^{-i\omega t}$. (In this case – as in the case of critically damped SHM discussed in Section 5.4 – you get only one solution this way. The other has the same form as in Eq. 5.43 for the second solution in damped SHM.) Write down the general solution. (c) At time $t = 0$, I push the puck from position $\mathbf{r}'_0 = (x'_0, 0)$ with velocity $\mathbf{v}'_0 = (v'_{x0}, v'_{y0})$ (all measured by me on the turntable). Show that

$$x'(t) = (x'_0 + v'_{x0}t)\cos\Omega t + (v'_{y0} + \Omega x'_0)t\sin\Omega t,$$

$$y'(t) = -(x'_0 + v'_{x0}t)\sin\Omega t + (v'_{y0} + \Omega x'_0)t\cos\Omega t.$$

(d) Describe and sketch the behavior of the puck for large values of t. [*Hint:* When t is large the terms proportional to t dominate (except in the case that both their coefficients are zero). With t large, write the above equations in the form $x'(t) = t(B_1\cos\Omega t + B_2\sin\Omega t)$, with a similar expression for $y(t)$. B now you can recognize that the path is the same kind of spiral, whatever the initial conditions (with the one exception mentioned.)]

2. **(10 points):** The "Flat Earth Society" has a website (www.theflatearthsociety.org) and in one discussion forum it is claimed that the motion of a Foucault pendulum is explained by the rotation of a flat earth about an axis through the North Pole. (a) Determine the precession rate of a Foucault pendulum on a flat earth. (b) From this result, could the flat–earthers have a point?

3. **(10 points):** A river runs northward along a channel of width d at a north latitude λ. Show that the height of the water on the east bank exceeds that on the west bank by $(2dv\Omega\sin\lambda)/g$, where v is the velocity of the water and Ω is the angular velocity of Earth. [Treat the water as flowing uniformly across the channel, neglecting

4. **(15 points):** The *Compton generator* is a beautiful demonstration of the coriolis force due to the earth's rotation, invented by the American physicist A.H. Compton (1892 – 1962, best known as author of the Compton effect) while he was still an undergraduate. A narrow glass tube in the shape of a torus or ring (radius R of the ring $\gg$ radius of the tube) is filled with water, plus some dust particles to let on see any motion of the water. The ring and water are initially stationary and horizontal, but the ring is then spun through 180° about its east-west diameter. Explain why this should cause the water to move around the tube. Show that the speed of the water just after the 180° turn should be $2\Omega R\cos\theta$, where Ω is the earth's angular velocity, and λ is the latitude of the experiment. What would this speed be if $R \approx 1\,\mathrm{m}$ and $\lambda = 50°$? Compton measured this speed with a microscope and got agreement within 3%.