PHY5210 **Problem Set #4** **due16 February 2015**

1. **(5 points):** A particle is confined to move on the surface of a circular cone with its axis on the z axis, vertex at the origin (pointing down), and half-angle α. The particle's position can be specified by two generalized coordinates, which you can choose to be the coordinates (ρ, ϕ) of cylindrical polar coordinates. Write down the equations that give the three Cartesian coordinates of the particle in terms of the generalized coordinates (ρ, ϕ) and vice versa.

2. **(5 points):** Figure 7.12 shows a crude model of a yoyo. A massless string is suspended vertically from a fixed point and the other end is wrapped several times around a uniform cylinder of mass m and radius R. When the cylinder is released it moves vertically down, rotating as the string unwinds. Write down the Lagrangian, using the distance x as your generalized coordinate. Find the Lagrange equation of motion and show that the cylinder accelerates downward with $\ddot{x} = 2g/3$. [*Hints:* Recall that the total kinetic energy of a body like the yoyo is $T = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$, where v is the velocity of the center of mass, I is the moment of inertia (for a uniform cylinder $I = \frac{1}{2}mR^2$) and ω is the angular velocity about the CM. You can express ω in terms of $\dot{x}$.]

3. **(10 points):** A ball of mass m rolls without slipping down a movable wedge of mass M. The angle of the wedge is θ, and it is free to slide on a smooth horizontal surface. Find the acceleration of the wedge.

4. **(15 points):** A bead of mass m is constrained to slide along a thin, circular hoop of radius ℓ that rotates with constant angular velocity ω in the xy plane about a point on its rim, as shown in the Figure 7.16 in Taylor. (a) Write down the Lagrangian for the bead. (b) Find Lagrange's equation of motion for the bead. (c) Show that the bead oscillates like a pendulum about the point on the rim diametrically opposite the point about which the hoop rotates. (d) What is the effective "length" of this "pendulum"?

5. **(15 points):** Consider the well-known problem of a cart of mass m moving along the x axis attached to a spring of spring constant k, whose other end is held fixed (see Figure 5.2 in Taylor). If we ignore the mass of the spring (as we almost always do) then we know that the cart executes simple harmonic motion with angular frequency $\omega = \sqrt{k/m}$. Using the Lagrangian approach, you can find the effect of the spring's mass M as follows: (a) Assuming that the spring is uniform and stretches uniformly, show that its kinetic energy is $\frac{1}{6}M\dot{x}^2$. (As usual x is the extension of the spring from its equilibrium length.) Write down the Lagrangian for the system of cart plus spring. (*Note:* The potential energy is still $\frac{1}{2}kx^2$.) (b) Write down the Lagrange equation and show that the cart still executes simple harmonic motion but with angular frequency $\omega = \sqrt{k/(m + M/3)}$; that is, the effect of the spring's mass M is just to add $M/3$ to the mass of the cart.