PHY5210 **Problem Set #5** **due 23 February 2015**

1. **(5 points):** Lagrange's equations in the form discussed in this chapter hold only if the forces (at least the nonconstraint forces) are derivable from a potential energy. To get an idea how they can be modified to include forces like friction, consider the following: A single particle in one dimension is subject to various conservative forces ($netconservativeforce = F = -\partial U/\partial x$) and a nonconservative force (let's call it F_{fric}). Define the Lagrangian as $\mathcal{L} = T - U$ and show that the appropriate modification is
$$\frac{\partial \mathcal{L}}{\partial x} + F_{\text{fric}} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}}.$$

2. **(10 points):** Find the differential equations of motion for a particle of mass m attached to an elastic string of spring constant k and unstretched length ℓ_0, free to swing in a vertical plane. This is a pendulum with a stretchy string.

3. **(10 points):** In this problem you will prove the equation of motion 9.34 for a rotating frame using the Lagrangian approach. As usual, the Lagrangian method is in many ways easier than the Newtonian (except that it calls for some vector gymnastics), but is perhaps less insightful. Let $\mathcal{O}'x'y'z'$ be a noninertial frame rotating with constant angular velocity Ω relative to the inertial frame $\mathcal{O}xyz$. Let the origins coincide, $\mathcal{O} = \mathcal{O}'$. (a) Find the Lagrangian $\mathcal{L} = T - U$ in terms of the coordinates $\mathbf{r}'$ and $\dot{\mathbf{r}}'$ of the noninertial frame. [Remember that you must first evaluate T in the inertial frame. Recall that $\mathbf{v} = \mathbf{v}' + \mathbf{\Omega} \times \mathbf{r}$.] (b) Show that the three Lagrange equations reproduce Eq. 9.34 precisely.

4. **(10 points):** A uniform solid hemisphere of radius R has its flat base in the xy plane with its center at the origin. Use the result of Taylor, 10.4 to find the center of mass.

5. **(15 points):** Automatic stabilization of the orientation of orbiting satellites utilizes the torque from the Earth's gravitational pull on a non-spherical satellite in a circular orbit of radius R. Consider a dumbbell-shaped satellite consisting of two point masses of mass m connected by a massless rod of length 2ℓ much less than R where the rod lies in the plane of the orbit (see Figure). The orientation of the satellite relative to the direction toward the Earth is measured by angle θ. Motion only occurs in the plane of the orbit.

   (a) Find the equilibrium positions and determine which is stable.

   (b) Show that the angular frequency of small-angle oscillations of the satellite about its stable orientation is $\sqrt{3}$ times the orbital angular velocity of the satellite.