PHY5210 **Problem Set #7** **due 9 March 2015**

1. **(10 points):** As an exercise in solving eigenvalue problems, find the principal moments and principal axes for an object whose inertia tensor in (x, y, z) coordinates is

$$\underline{\mathbf{I}} = \frac{ma^2}{16}\begin{pmatrix} 65 & 6 & 5\sqrt{3} \\ 6 & 52 & -2\sqrt{3} \\ 5\sqrt{3} & -2\sqrt{3} & 75 \end{pmatrix}$$

   Hint: One of the principal moments is $5ma^2$.

2. **(5 points):** In the weak coupling limit, the frequencies expressed in Eq. (11.27) are expressed as $\omega_1 = \omega_0 - \epsilon$ and $\omega_2 = \omega_0 + \epsilon$. By expanding (11.27) for $k_2 \ll k$, derive expressions for ω_0 and ϵ in terms of m, k, and k_2.

3. **(10 points):** A simple pendulum (mass M and length L) is suspended from a cart (mass m) that can oscillate on the end of a spring of force constant k, as shown in Figure 11.18. (a) Assuming that the angle ϕ remains small, write down the systems Lagrangian and the equations of motion for x and ϕ.
   (b) Assuming that $m = M = L = g = 1$ and $k = 2$ (all in appropriate units) find the normal frequencies, and for each normal frequency find and describe the motion of the corresponding normal mode.

4. **(15 points):** As a model of a linear triatomic molecule (such as CO_2), consider the system shown in Fig. 11.21 with two identical atoms each of mass m connected by two identical springs to a single atom of mass M. To simplify matters, assume that the system is confined to move in one dimension. (a) Write down the Lagrangian and find the normal frequencies of the system. Show that one of the normal frequencies is zero. (b) Find and describe the motion in the normal modes whose frequencies are nonzero. (c) Do the same for the mode with zero frequency. [*Hint:* See the comments at the end of Problem 11.27.]

5. **(10 points):** Consider the two coupled pendula of Problem 11.14. (a) What would be a natural choce for the normal coordinates ξ_1 and ξ_2?
   (b) Show that even if both pendula are subject to a resistive force of magnitude bv (with b small), the equations of motion for ξ_1 and ξ_2 are still uncoupled.
   (c) Find and describe the motion of the pendula for each mode.