PHY5210 **Problem Set #8** **due 30 March 2015**

1. **(5 points):** Consider a mass m constrained to move in a vertical line under the influence of gravity. Using the coordinate x measured vertically down from a convenient origin $\mathcal{O}$, write down the Lagrangian $\mathcal{L}$ and find the generalized momentum $p = \partial\mathcal{L}/\partial\dot{x}$. Find the Hamiltonian $\mathcal{H}$ as a function of x and p, and write down Hamilton's equations of motion. (It is too much to hope with a system this simple that you would learn anything new by using the Hamiltonian approach, but do check that the equations of motion make sense.)

2. **(10 points):** Write the Hamiltonian function and find Hamilton's canonical equations for the three–dimensional motion of a projectile in a uniform gravitational field with no air resistance. Show that these equations lead to the known result $m\ddot{\vec{r}} = \dot{\vec{p}} = -mg\hat{\mathbf{z}}$.

3. **(5 points):** Same as Problem 13.11 in Taylor, but use the following system: A bead of mass m is threaded on a frictionless, straight rod that lies in a horizontal plane and is fixed to spin with constant angular velocity ω about a vertical axis through the midpoint of the rod. Find the Hamiltonian for the bead and show that it is not equal to $T + U$.

4. **(15 points):** A particle of mass m is subject to a central, attractive force given by

$$\vec{F}(r,t) = -\hat{r}\frac{k}{r^2}e^{-\beta t}$$

where k and β are positive constants, t is the time, and r is the distance to the center of force. (a) Find the Hamiltonian function for the particle. (b) Compare the Hamiltonian to the total energy of the particle. (c) Is the energy of the particle conserved? Discuss.

5. **(15 points):** Two particles whose masses are m_1 and m_2 are connected by a massless spring of unstretched length ℓ and spring constant k. The system is free to move, rotate, and vibrate on top of a smooth horizontal plane that serves as its support. (a) Find the Hamiltonian of the system. Use the generalized coordinates X, Y, r, ϕ where (X, Y) is the position of the center of mass and (r, ϕ) are the polar coordinates of the position of m_1 relative to the center of mass. (b) Find the 8 Hamilton equations of motion. (c) What generalized momenta, if any, are conserved?