PHY5210 Problem Set #9 due 6 April 2015

1. **(5 points):** Here is a simple example of a canonical transformation that illustrates how the Hamiltonian formalism lets one mix up the q's and p's. Consider a system with one degree of freedom and Hamiltonian $\mathcal{H} = \mathcal{H}(q, p)$. The equations of motion are, of course, the usual Hamiltonian equations $\dot{q} = \partial\mathcal{H}/\partial p$ and $\dot{p} = -\partial\mathcal{H}/\partial q$. Now consider new coordinates in phase space defined as $Q = p$ and $P = -q$. Show that the equations of motion for the new coordinates Q and P are $\dot{Q} = \partial\mathcal{H}/\partial P$ and $\dot{P} = -\partial H/\partial Q$, that is, the Hamiltonian formulation applies equally to the new choice of coordinates where we have exchanged the roles of position and momentum.

2. **(10 points):** Figure 13.6 shows some phase–space orbits for a mass in free fall. The points A_0, B_0, C_0, D_0 represent four different possible initial states at time 0, and A, B, C, D are the corresponding states at a later time. Write down the position $x(t)$ and momentum $p(t)$ as functions of t and use these to prove that $ABCD$ is a parallelogram with area equal to the rectangle $A_0B_0C_0D_0$. [This is an example of Liouville's theorem.]

3. **(10 points):** The cross section for scattering a certain nuclear particle by a copper nucleus is 2.0 barns. If 10^9 of these particles are fired through a copper foil of thickness $10\,\mu$m, how many particles are scattered? (Copper's density is $8.9\,\text{grams/cm}^3$ and its atomic mass is 63.5. The scattering by any atomic electron is completely negligible.)

4. **(5 points):** In their famous experiment, Rutherford's assistants, Geiger and Marsden, detected the scattered alpha particles using a zinc supplied screen, which produced a tiny flash of light when struck by an alpha particle. If their screen had area $1\,\text{mm}^2$ and was 1 cm from the target, what solid angle did it subtend?

5. **(10 points):** The differential cross section for scattering 6.5 MeV alpha particles at 120° off a silver nucleus is about 0.50 barns/sr. If a total of 10^{10} alphas impinge on a silver foil of thickness $1.0\,\mu$m and if we detect the scattered particles using a counter of area $0.10\,\text{mm}^2$ at 120° and 1.0 cm from the target, about how many scattered alphas should we expect to count? (Silver has a specific gravity of 10.5, and atomic mass of 108.)

6. **(10 points):** One can set up a two–dimensional scattering theory, which could be applied to puck projectiles sliding on an ice rink and colliding with various target obstacles. The cross section σ is the effective width of a target, and the differential cross section $d\sigma/d\theta$ gives the number of projectiles scattered in the angle $d\theta$. (a) Show that the two–dimensional analog of (14.23) is $d\sigma/d\theta = |d\sigma/d\theta|$. (Note that in two–dimensional scattering it is convenient to let θ range from $-\pi$ to π.) (b) Now consider the scattering of a small projectile off a hard "sphere" (actually a hard disk) of radius R fixed to the ice. Find the differential cross section. (Note that in two dimensions, hard "sphere" scattering is not isotropic.) (c) By integrating your answer to part (b), show that the total cross section is $2R$ as expected.