PHY5210 **Problem Set #10** **due 13 April 2015**

1. **(10 points):** One of the first observations that suggested the nuclear model of the atom to Rutherford was that several alpha particles scattered by metal foils emerged in the backward hemisphere, $\pi/2 \leq \theta \leq \pi$, an observation that was impossible to explain on the basis of rival atomic models, but emerged naturally from the nuclear model. In an early experiment, Geiger and Marsden measured the fraction of incident alphas scattered into the backward hemisphere off a platinum foil. By integrating the Rutherford cross section (14.33) over the backward hemisphere, show that the cross section for scattering with $\theta \geq \pi/2$ is $4\pi\sigma_0(E)$. Using the following numbers, predict the ratio of $N_{\rm sc}(\theta \geq \pi/2)/N_{\rm inc}$: thickness of platinum foil of $3\,\mu$m, density of $21.4\,{\rm g/cm}^3$, atomic weight of 195, atomic number of 78, and energy of incident alphas of 7.8 MeV. Compare your result with their estimate that "of the incident α particles about 1 in 8000 is reflected" (that is, scattered into the backward hemisphere). Small as this fraction is, it is fa larger than the rival models of the atom could explain.
2. **(10 points):** When the LHC restarts in 2015, beams of protons with accelerated to an energy of $6.5\,{\rm TeV} = 6500\,{\rm GeV}$ will collide. (a) Given that the mass of the proton is 0.938 GeV, determine the difference between the speed of the protons in the LHC and the speed of light. Express the result in units of kilometer per hour. (b) The protons are accelerated to the collision energy (and maintained there in spite of small losses of energy due to radiation) with a electric fields that pulsate in time with the passage of bunches of protons. The timing of the pulsations is calculated based on the speed of the protons calculated in (a). Assume that the laws of relativity are wrong, and calculate the speed of the protons assuming $E = \frac{1}{2}mv^2$. Given this result, is it possible that bunches of protons can be stored in a 27 km orbit for tens of hours?
3. **(10 points):** The pion (π^+ or π^-) is an unstable particle that decays with a proper half-life of 1.8×10^{-8} s. (This is the half-life measured in the pion's rest frame.) (a) What is the pion's half-life measured in a frame $\mathcal{S}$ where it is traveling at $0.8c$? (b) If 32,000 pions are created at the same place, all traveling at this same speed, how many will remain after they have traveled down an evacuated pipe of length $d = 36\,$m? Remember that after n half-lives, 2^{-n} of the original particles survive. (c) What would the answer have been if you had ignored time dilation? (Naturally it is the answer (b) that agrees with experiment.)
4. **(10 points):** A GPS satellite is in a circular orbit at an altitude of 12,200 km above the Earth. (a) What is the orbital speed and γ factor for the satellite? (b) As observed from the ground, by how much does the clock on the satellite differ from a ground–based clock after one hour (as measured on the ground)? What is the percent difference? (c) The same clocks will also run differently due to general relativistic effects. According to general relativity, time slows down with gravity like $t_g = t_{no\ g}\sqrt{1 - 2rg/c^2}$ where g is the local acceleration due to gravity. What is the time difference between the clocks due to gravitational time dilation? Be careful to get the sign correct. (d) What is the total time difference due to both effects. Convert the time difference into a distance to estimate how far off the GPS system would be without correcting for relativistic effects after one hour.
5. **(10 points):** Let $\underline{\mathbf{\Lambda}}_B(\theta)$ denote the 4×4 matrix that gives a pure boost in the direction that makes an angle θ with the x_1 axis in the x_1x_2 plane. Explain why this can be found as $\underline{\mathbf{\Lambda}}_B(\theta) = \underline{\mathbf{\Lambda}}_R(-\theta)\underline{\mathbf{\Lambda}}_B(0)\underline{\mathbf{\Lambda}}_R(\theta)$, where $\underline{\mathbf{\Lambda}}_R(\theta)$ denotes the matrix that rotates the x_1x_2 plane through angle θ and $\underline{\mathbf{\Lambda}}_B(0)$ is the standard boost along the x_1 axis. Use this result to find $\underline{\mathbf{\Lambda}}_B(\theta)$ and check your result by finding the motion of the spatial origin of the frame $\mathcal{S}$ as observed in $\mathcal{S}'$.