PHY5210 **Problem Set #11** **due 20 April 2015**

1. **(5 points):** We have seen that the scalar product $x \cdot x$ of any four-vector x with itself is invariant under Lorentz transformations. Use the invariance of $x \cdot x$ to prove that the scalar product of $x \cdot y$ of any two four-vectors x and y is likewise invariant.
2. **(5 points):** Particle A (energy E) hits particle B (at rest), producing particles $C_1, C_2, \ldots$: $AB \to C_1 C_2 \ldots C_n$. Show that the threshold (*i.e.* minimum E) for this reaction in terms of the particle masses is
$$E_{th} = \frac{M^2 - m_A^2 - m_B^2}{2m_B} c^2 ,$$
where $M = m_1 + m_2 + \cdots + m_n$.
3. **(10 points):** A pion traveling at speed $v = \beta c$ decays into a muon and a neutrino: $\pi^- \to \mu^- \bar{\nu}_\mu$. If the neutrino emerges at $90°$ to the original pion direction, show that the μ emerges at angle θ where $\tan\theta = (1 - m_\mu^2/m_\pi^2)/(2\beta\gamma^2)$. The neutrino has an extremely small mass and can be assumed to be massless for this problem.
4. **(5 points):** A particle of mass M decays at rest into two identical particles each of mass m. Use conservation of momentum and energy to find the speed of the outgoing particles.
5. **(5 points):** Two balls of equal masses m approach one another head-on with equal but opposite velocities of magnitude $0.8c$. Their collision is perfectly inelastic, so they stick together and form a single body of mass M. What is the velocity of the final body and what is its mass M?
6. **(10 points):** A mass m is thrown from the origin at $t = 0$ with initial three-momentum $\mathbf{p}_0 = p_0\hat{\mathbf{y}}$. If it is subject to a constant force $\mathbf{F} = F_0\hat{\mathbf{x}}$, find its velocity $\mathbf{v}$ as a function of t, and by integrating $\mathbf{v}$ find its trajectory (using full relativistic formulas). Check that in the nonrelativistic limit the trajectory is the expected parabola.
7. **(10 points):** The factor γ in the Doppler formula (15.64), which can be ascribed to time dilation, means that even when $\theta = 90°$ there is a Doppler shift. (In classical physics there is no Doppler shift when $\theta = 90°$ and the source has zero velocity in the direction of the observer.) This *transverse Doppler shift* is therefore a test of time dilation, and has yielded some very accurate tests of the theory. However, except when the source is moving very close to the speed of light, the transverse shift is quite small. (a) If $V = 0.2c$, what is the percentage shift for $\theta = 90°$? (b) Compare this with the shift when the source approaches the observer head-on.

8. **(extra credit 10 points):** Consider a one-dimensional, relativistic harmonic oscillator for which the Lagrangian is

$$\mathcal{L} = mc^2\left(1 - \sqrt{1-\beta^2}\right) - \frac{1}{2}kx^2.$$

Obtain the Lagrange equation of motion and show that it can be integrated to yield $E = mc^2 + \frac{1}{2}ka^2$ where a is the maximum excursion from equilibrium of the oscillating particle. Show that the period

$$T = 4\int_{x=0}^{x=a} dt$$

can be expressed as

$$T = \frac{2a}{\kappa c}\int_0^{\pi/2} \frac{1+2\kappa^2\cos^2\phi}{\sqrt{1+\kappa^2\cos^2\phi}}d\phi.$$

Expand the integrand in powers of $\kappa = (a/2)\sqrt{k/mc^2}$ and show that, to first order in κ,

$$T \approx T_0\left(1 + \frac{3}{16}\frac{ka^2}{mc^2}\right)$$

where T_0 is the nonrelativistic period for small oscillations, $2\pi\sqrt{m/k}$.