PHY5210 **Problem Set #12** **due 27 April 2015**

1. **(5 points):** Verify that the quantity $c = \sqrt{\tau/\mu}$ that appears in the wave equation for a string has the units of speed.

2. **(5 points):** Let $f(\xi)$ be an arbitrary, twice differentiable function. Show by direct substitution that $f(x - ct)$ is a solution of the wave equation.

3. **(5 points):** Show that if we make the change of variables $\xi = x - ct$ and $\eta = x + ct$, then

$$\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = -4c^2 \frac{\partial}{\partial \xi} \frac{\partial u}{\partial \eta}.$$

4. **(10 points):** Let $f(\vec{r})$ be any spherically symmetric function, that is, when expressed in spherical polar coordinates, (r, θ, ϕ), it has the form $f(\vec{r}) = f(r)$, independent of θ and ϕ. (a) Starting from the definition of $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$, prove that

$$\nabla^2 f = \frac{1}{r} \frac{\partial^2}{\partial r^2} (rf).$$

   (b) Prove the same result using the formula inside the back cover for ∇^2 in spherical polar coordinates. [Obviously, this second proof is much simpler, but the hard work is hidden in the derivation of the formula for ∇^2.]

5. **(15 points):** Show that if the stress tensor $\underline{\mathbf{\Sigma}}$ is diagonal (all off–diagonal elements zero) with respect to *any* choice of orthogonal axes, then it is in fact a multiple of the unit matrix. This gives an alternative and elegant proof that if there are no shearing stresses (in any coordinate system) then the pressure is isotropic. To do this problem, you need to know how the elements of a tensor transform as we rotate the coordinate axes, as described in Section 15.17. Assume that with respect to one set of axes $\underline{\mathbf{\Sigma}}$ is diagonal but that not all three diagonal elements are equal. (For example, $\sigma_{11} \neq \sigma_{33}$.) It is not hard to come up with a rotation — that of Equation (15.36) will do — such that in the rotated system $\sigma'_{13} \neq 0$.

6. **(10 points):** Equations (16.129) and (16.130) are two different forms of the equation of continuity. Prove that they are equivalent.